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ORIGINAL ARTICLE

Developing primary pre-service teachers' mathematical content knowledge: opportunities and influences

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Abstract
There is a consensus that we need to improve the quality of pre-service teacher education, and teachers' mathematical content knowledge is critical for teaching. Identifying opportunities and influences that assist pre-service teachers to extend their mathematical content knowledge throughout their teacher education programme is important. This paper reports on qualitative data, collected over 4 years from two typical pre-service teachers whose developing mathematical content knowledge was investigated during their primary and secondary programme. These data were analysed and reported using the four dimensions of the Knowledge Quartet: foundation knowledge, transformation, connection and contingency. The results highlight the consequences of programme structure in order to help pre-service teachers to establish and sustain a positive mathematics learner identity, build teacher identity and develop mathematical content knowledge.

Keywords Pre-service teachers · Mathematics · Mathematical content knowledge · Primary · Teacher identity · Foundation knowledge · Practicum · Coursework

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Introduction

It is well documented that teachers use and need different types of mathematical knowledge for teaching mathematics to primary students. Ma (1999) in her research of effective teachers of mathematics distinguished key elements of a teachers' knowledge: breadth and depth. Higher education providers in Australia have also advised of the need to lift student outcomes by ensuring [beginning] teachers demonstrate depth of mathematical content knowledge (MCK) (Teacher Education Ministerial Advisory Group [TEMAG] 2014). Others agree that proficient teachers require knowing school mathematics with breadth and depth including a "broad and connected knowledge of the content at hand … an understanding of big ideas … knowledge of effective ways to introduce students to particular mathematical ideas, and ways to instil understanding" (Schoenfeld and Kilpatrcik 2008, p. 327).

Pre-service teachers (PSTs) develop the knowledge and skills as beginning teachers during their teacher education programme. However, how different programmes influence the development of mathematics teachers' knowledge is unclear (Tatto et al. 2012). MCK is a critical attribute for teaching mathematics and underpins the decisions teachers make when teaching (Rowland et al. 2009). Teacher education should ensure that PSTs are offered a range of contexts to develop and improve their MCK (Callingham et al. 2012). Therefore, a longitudinal study was designed to identify opportunities and influences contributing to the development of PSTs' MCK aimed to inform quality practices in future education programmes.

Literature review

During mathematics lessons, a teacher's mathematical knowledge for teaching influences the quality of their instruction (Ball et al. 2008; Beswick 2012; Mapolelo and Akinsola 2015). As well as breadth and depth of knowledge (Ma 1999; Schoenfeld and Kilpatrcik 2008), highly effective teachers of mathematics have specific characteristics regarding knowledge, beliefs and practices. For example, they make connections between different mathematical ideas; can select and use efficient strategies when teaching; address misconceptions and use mathematical language during instruction (Askew et al. 1997).

Rather than reporting on how teachers acquire their MCK, studies have tended to focus on deficiencies in teachers' mathematics knowledge (e.g., Ma 1999; Ponte and Chapman 2006; Thames and Ball 2010). A study designed to investigate how PSTs with weaknesses in their MCK at the start of their programme develop MCK during teacher education will contribute to the literature. Identifying how teachers attain their knowledge for teaching is important and can assist with improving mathematics teacher education (Anthony et al. 2012; Anthony et al. 2016).

Many Australian studies of PST education have been short-term and/or focused on one type of data collection rather than multiple types of data, for example, data collection from multiple choice tests or questionnaires (Callingham et al. 2012). Whilst written tests are useful for larger studies, interviews would be more likely to provide higher quality information about teachers' (and PSTs') understanding of MCK (Chick 2003). Clements (2008) in his review of mathematics education research agreed and recommended that multiple data collection methods would strengthen future studies.

Kulm (2008) suggested that improvement of teacher preparation must be grounded in research. Others agreed there is also a need for further understanding of how PSTs use knowledge in practice with students (Anthony et al. 2012; Anthony et al. 2016; Ball and Even 2009) with few studies focussing on PSTs' development of MCK throughout all aspects of their programme including practicum teaching experiences and coursework.

Mathematical knowledge for teaching

Since Shulman's (1987) seminal study highlighting seven categories of a teacher's knowledge, including content knowledge, other scholars have contributed to the understanding of the types of knowledge for teaching mathematics. Ma (1999) described teachers who demonstrate conceptual understanding of mathematics as having profound understanding of mathematics including evidence of "breadth, depth and thoroughness" (p. 124). Other frameworks have been designed to extend our understanding of the special knowledge a teacher uses and needs when teaching mathematics. Ball et al. (2008) described specialised content knowledge (SCK) as a unique kind of knowledge specific to teachers.

The Knowledge Quartet has been used in the past to assist teachers including primary PSTs to analyse their mathematics teaching with a focus on MCK for teaching (Rowland et al. 2009; Thwaites et al. 2011; Turner 2012). The Knowledge Quartet was suitable for this study, providing four dimensions: *foundation knowledge*; *transformation*; *connection*; and *contingency* (Table 1) and codes that could assist with deepening understanding of how these important dimensions might be developed during teacher education.

The four dimensions (or categories) and codes shown in Table 1 also draw on categories within other theoretical frameworks and mathematical knowledge for

Table 1 The dimensions of the Knowledge Quartet (Rowland et al. 2009, p. 29)

Dimension	Codes
Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Choice of examples Choice of representation Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between concepts Making connections between procedures Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities

teaching. *Foundation knowledge* includes overt subject knowledge and pedagogical content knowledge (PCK) (Shulman 1987), common content knowledge and SCK (Ball et al. 2008) as well as beliefs (Rowland et al. 2009). *Transformation* relates to PCK (Shulman 1987) comprising knowledge of content and pedagogy, which are important when teachers choose tasks to facilitate student learning (Ball et al. 2008). *Connections* relate to the coherence of planning across a lesson or series of lessons (Rowland et al. 2009) and connections to the curriculum and hence knowledge of curriculum (Ball et al. 2008; Shulman 1987). The fourth category, *Contingency*, concerns teachers' responses to unexpected events during teaching (Rowland et al. 2009).

Foundation knowledge also includes numeracy skills such as having an accurate understanding of mathematical ideas or concepts (Rowland et al. 2009). At the time of data collection, PSTs were not required to pass the Literacy and Numeracy Test for Initial Teacher Education (LANTITE). The Literacy and Numeracy Test is an online assessment tool designed to assess PSTs' personal literacy and numeracy skills and now, must be passed prior to graduation (Australian Council for Educational Research [ACER] 2015). Nevertheless, the findings of this study are important when identifying opportunities that enhanced PSTs' MCK including their *foundation knowledge*.

Factors influencing mathematical knowledge for teaching

Teacher identity is developed over time as teachers consider how to be, how to act and how to understand their work (Sachs 2005). There may be a number of factors that influence PSTs' development of their MCK including teacher identity. Walshaw's (2008) theory of learning to teach recognised the construction of identity as important, influenced by a PST's past educational experiences as a student and teacher education programmes including coursework and practicum experiences. Teacher identity may be influenced by attitudes, beliefs and perceptions of teaching mathematics as well as their teacher education experiences (McDonald et al. 2014). Identity as a mathematics thinker can develop through self-directed learning using a combination of learning experiences (Owens 2007/2008).

Pre-programme identity may also influence the process of becoming a teacher and development of PSTs' MCK. Pre-programme identity includes prior beliefs about mathematics teaching and learning mathematics at school (Anthony et al. 2012) or self-efficacy, shaping PSTs' perceptions about how mathematics is taught (Hodgen and Askew 2007; Walshaw 2008) and how they respond to opportunities to learn (Tatto et al. 2009). Pre-programme identity may include a reliance on procedural methods developed at school before commencing teacher education (Ponte and Chapman 2008) or beliefs such as "mathematics is difficult" or "mathematics is all about one answer" (Liljedahl 2005, p. 1). Self-efficacy may influence the choices individuals make; how they engage in tasks; the length of time they persist on tasks or avoidance of tasks if they are less confident (Pajares 1996). Emphasis on PSTs' development of teacher identity, that is, "assuming the values and norms of the profession" (Ponte and Chapman 2008, p. 241), may be a means of fostering knowledge for teaching mathematics (Van Zoest and Bohl 2005).

During teacher education mathematics, teacher identity would be realising the importance of *foundation knowledge*, learning to *transform* this knowledge for teaching, making *connections* and considering how to deal with *contingencies*. Teacher identity related to mathematics teaching would be developed when participating in coursework experiences, attending lectures, participating in tutorials and responding to assignments. School placement experiences would also be important for developing a PST's identity as a mathematics teacher when teaching with their mentor/supervising teacher.

The study

This study was designed to identify how PSTs extend their knowledge and skills, specifically their MCK. Qualitative and quantitative methods were used to identify PSTs' readiness for teaching primary students, including how and when PSTs learnt MCK.

This longitudinal study will supplement the many studies of PSTs' MCK reported over short timeframes during different stages of PSTs' programmes (Adler et al. 2005; Anthony et al. 2012) extending our understanding of when and how PSTs develop MCK. The findings may also provide possibilities for consideration for improving the structure of future teacher education programmes, especially those qualifying teachers with primary and secondary degrees.

In this article, we report the findings for two PSTs from the larger longitudinal study (Livy 2014). Using multiple methods of data collection to analyse both coursework and practicum experiences, this article reports on their MCK and how they made sense of their *foundation knowledge*, developed and used *transformations*, made *connections* and used opportunities to respond to *contingencies*. The study was guided by the following research question:

> What opportunities and influences enhanced the development of two PSTs' MCK including: *foundation knowledge*, *transformation*, *connection*, and *contingency* during a four-year primary and secondary teacher education program?

The next section will outline the research design, context and methods used in this study.

Research design

Qualitative and quantitative methods were chosen to report on a 4-year longitudinal study of two PSTs and opportunities and influences that enhanced development of their MCK. An ethnographic approach was chosen because the researchers were interested in understanding as much as possible about the setting where the study took place as well as the capacity to reflect on longitudinal trends (Kervin et al. 2006).

Setting and programme structure

The participants were completing a 4-year Bachelor of Education programme (preparatory, year 12), gaining qualifications to teach primary students, including

Table 2 Programme structure

Year	Core mathematics education units	Elective mathematics education units	Teaching experience
1	Unit 1A 12 × (2 h workshops and 1 h lecture)	Unit 1B 12 × 2 h workshops	Primary school (20 days)
2	Unit 2A 12 × 2 h workshops Unit 2B 12 × (2 h workshops and 1 h lecture)	Unit 1B*	Primary school (32 days)
3	None	Unit 1B* Secondary discipline teaching units	Secondary school—discipline specialisation (42 days)
4	None		Primary school (50 days)

*Compulsory for PSTs who did not pass MCSK test during year 2

mathematics, and two secondary school specialisation subjects. Table 2 shows when these two PSTs completed units related to primary mathematics education and the number of days of school-based teaching experience during each year of the programme.

The programme included three core primary mathematics education units (Units 1A, 2A and 2B). However, some PSTs completed an additional unit if needed (Unit 1B) providing another opportunity to pass the Mathematical Competency Skills and Knowledge (MCSK).

As part of the programme requirements, all PSTs were required to pass an internal MCSK test. The purpose of the test was to ensure PSTs demonstrated MCK of items examining understanding of year 5 to year 8 mathematics (e.g., ACARA 2017). This was also in line with the Australian Institute and National Standards stating that graduating "teachers know the content and how to teach it" (Australian Institute for Teaching and School Leadership [AITSL] 2017, p. 3).

The MCSK test consisted of 7 sections, 7 items in each section, including closed- and open-question types, assessing mathematical knowledge (MCK and PCK items). More sections were related to the domain of number, including common and decimal fractions; other items assessed knowledge of geometry and measurement with fewer items assessing knowledge of statistics. PSTs were required to correctly respond to 5 of the 7 items in each section to gain a pass. If PSTs had difficulty passing the test, they were required to enrol in Unit 1B designed to extend their MCK and other knowledge for teaching primary mathematics. Sometimes, PSTs completed Unit 1B more than once.

PSTs also completed 100 days of practicum placement in primary schools (occurring in first, second and fourth years) and 45 days in a secondary school (in third year and related to their secondary specialisation). This number of days far exceeded the teacher education programme accreditation minimum of 45 days (Victorian Institute of Teaching (VIT) 2010). PSTs were usually assigned to one school for each year of the programme, attending their school once a week with additional weekly blocks as reported in the results.

Selection of participants

Lisa and Rose (pseudonyms) were selected for this study and chosen from 17 PSTs who participated in a longitudinal study during their teacher education programme (Livy 2014). Of concern, at the university where the study took place, was the significant proportion of PSTs who had difficulty passing Unit 1B including the MCSK test. Lisa was chosen as an example of a PST who had difficulty passing the MCSK test (during Units 1B and 2B), taking three attempts, and therefore was representative of one third of her peers (29%) in the larger study. Rose commenced her programme with concerns relating to her MCK. However, after completing a practice MCSK test in year 1, Rose acknowledged that she had difficulty responding to the test items and elected to enrol in Unit 1B at the end of her first year. Rose passed the MCSK test and Unit 1B before commencing year 2. Therefore, Lisa and Rose were chosen because they were considered typical of beginning PSTs in the larger study (and programme) as they both were required to develop and extend their MCK in order to pass programme requirements.

Data collection and analysis

Multiple types of data were collected to identify opportunities and influences that enhanced the development of Lisa's and Rose's MCK. Qualitative data included assignments, lesson plans, field notes and audio recordings of interviews and lessons. Quantitative data included PSTs' scores when responding to the MCSK test, but was not reported in detail as part of this paper. Semi-structured interviews were completed after the first author had observed a lesson. The interviews had two main purposes. They were designed to gather additional reflections from the PSTs, including data about

Table 3 Data gathering and analysis

Method	Data gathered	Analysis
Qualitative	Artifacts: All assignments submitted in Units 1A, 1B, 2A and 2B Primary mathematics lesson plans year 2 and year 4	Data uploaded into NVivo and coded using the Knowledge Quartet (Rowland et al. 2009)
Qualitative	Responses to questionnaire (year 2)	Descriptive analysis to determine demographic factors Open coding to identify prior learning, beliefs, attitudes and identity
Qualitative	Lesson observations (years 2 and 4)	Interviews and lesson observations transcribed and data uploaded into NVivo and coded using the Knowledge Quartet (Rowland et al. 2009)
Qualitative	Interviews (years 2, 3 and 4)	Open coding to identify identities, opportunities to develop MCK or constraints on developing MCK
Quantitative	MCSK test	Scores reported using descriptive statistics

opportunities, experiences and influences that enhanced their MCK during primary mathematics coursework and teaching practice experiences. The second purpose was to discuss the lesson the researcher had just observed, specifically identifying what was involved in planning the lesson and whether the pre-service teacher needed to check or revise their MCK before teaching the lesson.

Table 3 shows methods of data collection and analysis.

Data collection (Table 3) and analysis were ongoing and occurred throughout each year of the study when the PSTs were in schools. Placement occurred in weekly blocks as well as regularly on a Tuesday throughout the semester. Therefore, data was collected during different times between May and September depending on when it suited the schools, mentor teacher and PSTs.

All data were entered into NVivo. To manage the large amount of data, content analysis using inductive coding (Simminoff and Jacoby 2008) was suitable for identifying themes and discussing the findings including opportunities or influences that enhanced or hindered pre-service teachers' MCK. Learner identity, including identity as learners of mathematics and self-efficacy, was inferred from the ways PSTs chose to extend their *foundation knowledge* and was shaped by programme choices. Teacher identity was inferred from the ways in which PSTs realised the importance of implementing and developing dimensions of the Knowledge Quartet. Nodes and hierarchical coding were used to group the data, for example, by year of programme, assignments and PST interview data. Next, the four dimensions and codes of the Knowledge Quartet (Rowland et al. 2009) (Table 1) were used to code data including coursework tasks, practicum experiences, lesson observation field notes and interview responses. Discussion of coding occurred and was continued until agreement was reached by all authors.

Results

The results are reported chronologically and according to programme component to clearly present when and how Lisa and Rose developed their MCK during the programme. Evidence of development of their *foundation knowledge*, *transformation*, *connection* and *contingency* is reported. Similarities and differences between Rose's and Lisa's experiences reveal patterns and common themes that identified opportunities and factors influencing the development of their MCK. The results are presented according to each stage of the programme: pre-programme identity; primary mathematics coursework during year 1 and year 2 and primary mathematics practicum during

Table 4 Pre-programme identity regarding mathematics

Lisa	Rose
I [just] passed year 12 maths … I remember in primary school I felt a sense of fear when the teacher said get out your maths books. I wasn't even bad at it. I just didn't feel confident.	Although I made it to year 11 and the start of year 12 Maths Methods, I wasn't confident and wasn't clear as to why I did what I had to do to get the answer ... my maths experience was quite negative and unclear. I was taught only the rules.

year 1, year 2 and year 4. Third year is excluded because the programme focus was on secondary specialist subject teaching and practicum rather than primary teacher education. Each stage includes a table comparing Lisa's and Rose's experiences, followed by the categorisation of the experiences according to the Knowledge Quartet.

Pre-programme identity

Table 4 shows a comparison of Lisa's and Rose's interview reflections on their experiences of learning mathematics during primary and secondary school.

Both of these comments were coded as *pre-program identity* because they related to memories of learning mathematics at school. Lisa and Rose commenced the programme with similar pre-programme identity. They had negative experiences and both lacked confidence when learning mathematics. These attitudes indicate that they may also have difficulty relying on their *foundation knowledge* when planning and teaching mathematics. Rose, however, did show evidence of wanting to make sense of mathematics.

Primary mathematics coursework experiences

In this section, the coursework experiences are considered. Lisa and Rose completed core primary mathematics Units 1A, 2A and 2B during the first 2 years of the programme. Lisa and Rose both reported that they elected to study Unit 1B during year 1 because they wanted to build their understanding of mathematical skills, including categories of *foundation knowledge*. A hurdle task, the MCSK test, for Unit 1B (and/or Unit 2B) was undertaken to assess PSTs' *foundation knowledge*. Lisa required three attempts before passing the MCSK test, in year 1 (Unit 1B), year 2 (Unit 2B) and passed when repeating Unit 1B in year 4. Rose passed the MCSK test in Unit 1B during year 1.

Year 1 focused on developing different aspects of PSTs' *foundation knowledge* including evidence of *awareness of purpose*, beliefs about how and why mathematics is learnt and evidence of understanding mathematical concepts and ideas.

Unit 1A introduced PSTs to primary mathematics teaching and focused on their personal learning. Lisa and Rose provided comments on the second assignment, a self-directed learning task of a mathematical topic requiring improvement (Table 5).

Table 5 Foundation knowledge (Unit 1A self-directed investigation, year 1)

Lisa	Rose
Mine [investigation] was about water usage. In my personal learning plan [goals], I wrote I needed help with statistics, interpreting graphs, decimals and fractions. So, I worked out the average water usage and looked at past year's rainfall. My main aim was representing information in graphs and understanding fractions better. It didn't really help me understand my weakness … I just did it and did not get much out of it.	We did the maths investigation task that was the most useful for me. I reported on a trip to Italy measuring the distance for a ferry ride from one place to another place. I had like measurement in it calculating the cost for everything. They wanted us to use as much maths as possible.

The self-directed learning task was coded as *foundation knowledge - concentration on procedures* because the focus of the task was to help develop PSTs' MCK. Lisa focused on extending her understanding of statistics, and Rose chose an investigation that would help her practice calculating with decimals for measurement and costing.

A review of Lisa's investigation revealed limited mathematics. Rather than sourcing data and including her own graphs when reporting on accumulated rainfall of Melbourne, Lisa chose to source the graphs from the internet including these in her assignment. However, Lisa was able to interpret and write about these, coded as *foundation knowledge - awareness of purpose*. Rose chose measurement for her topic and completed a number of written calculations (multiplying with decimals) when working out the cost of her travel expenses for a holiday to Italy, coded as *foundation knowledge - concentrating on procedures*.

Table 6 shows comments regarding why Lisa and Rose chose to study Unit 1B after completing Unit 1A and reflections of these experiences.

Lisa's year-1 coursework experiences assisted her to identify her need to extend her *foundation knowledge*. She was able to consider how she might do this—use a textbook and enrol in Unit 1B. Lisa was starting to identify herself as a teacher of mathematics realising her need for *foundation knowledge* providing evidence of *awareness of purpose*. However, she still had difficulty in shifting her negative learner identity to act on this *awareness of purpose*.

Rose demonstrated a more positive learning identity and developing teacher identity when completing Unit 1B. She described how she actively and strategically revised her *foundation knowledge* by doing the mathematics as part of her coursework and on her own. Rose also highlights how Unit 1B focused on hands-on experiences, to develop understanding rather than relying on procedures, supporting the development of *foundation knowledge*. Rose's comments illustrate she was making *connections*, thus making sense of her mathematics and *transforming* her knowledge by learning different

Table 6 Learner identity (Unit 1B MCSK test, year 1)

Lisa	Rose
In first year, I took up the extra numeracy subject they offered [because] they gave us a practice test. I couldn't get through it. I think there is a long way to go. Just revision really. I haven't had time to do that yet. The unit didn't teach me so much about how to teach but taught me all the stuff I had forgotten. I knew that I was going to fail the first [MCSK] test so I didn't study and I failed. Now I am in second-year and teaching I realise that I should have done more last year rather than just sort of drift along.	By doing this unit and studying for the test helped to revise my maths. I went away in my own time and researched all the areas I had difficulty with. The learning log was really helpful and it sort of forced you to study. I just went through all the tests and I would go back and look at things and then I would research how they got the answers, I would ask my lecturer if I got stuck. The program made me refresh it and I just feel so much better. I did summer school maths as I did not want to have that and the education maths units at the same time. I could just focus on maths and it was on the holidays and I had plenty of time to study. It was about hands on experiences. Doing it ourselves physically rather than just doing it on paper was the most helpful in developing my understanding of maths.

ways to represent the mathematics, in particular when preparing for, and passing the MCSK test.

Year-2 coursework

Year-2 coursework focused on developing MCK, and PCK, knowledge of the primary mathematics curriculum and assessment. In Unit 2A, PSTs conducted the Early Years Numeracy Interview (Department of Education Employment and Training [DEET] 2001), assessed then reported on one student's response to the interview items. For the second assignment, PSTs worked in pairs to plan and teach a series of five mathematics lessons, one per week for 5 weeks, with a small group of students at a local primary school which was not their practicum school. For Unit 2B, PSTs worked in groups of three to plan and prepare a workshop presentation on a mathematics topic to their peers at university. They designed a learning trajectory of a mathematical topic such as fractions or decimals including the stages of learning and links to the primary mathematics curriculum. They also constructed five high-quality primary mathematics-teaching resources. During their interview in year 2, Lisa and Rose reflected on some of these experience (Table 7). Table 7 also includes excerpts from their Early Years Numeracy Interview assignment.

Lisa described planning a series of fraction lessons which were coded as *concentrating on procedures* and *making connections* including *decisions about sequencing*. As Lisa had to relearn some of the content, she was also revising her *foundation knowledge*. When reflecting on Charlie's understanding and use of repeated addition,

Table 7 Emerging teacher identity (year 2)

Lisa	Rose
… we are doing fractions... every week we plan and evaluate our lesson. We just reflected [with their group of students] on numerator and denominator. Then we talked about proper and improper, the next week we went into equivalent fractions and mixed numbers. Week three we worked adding and subtracting and we plan to cover a bit of a recap with that and finding common factors for adding subtracting fractions. It is Year 6 and I had to relearn some of it. I had forgotten how to turn it into a mixed number. We read Booker [program textbook] and that helped a lot. I feel more confident there than when I am at my placement school.	My understanding of mathematics has changed a lot since doing these subjects. I had to think why did I do that and how would I explain that to someone else. [My thinking] is completely different. The program has changed how I think about maths. Fractions, I did [presented] a workshop on fractions [Unit 2B] so it made me go away and research more. Area and volume, I found the lecture and workshop very clear and I kind of already had an idea of that as well. I memorised the names of shapes but now it is fading and I would have to research it again.
Charlie was able to solve mathematical problems mentally by using doubles and repeated addition to solve the equation. Charlie used the repeated addition method to solve the multiplication problem. (Lisa, numeracy report, p. 1)	Molly was able to use her fingers and count backwards with the one digit task efficiently, which shows that she relies on her fingers to solve problems and has to use one to one correspondence… yet when asked a more complex worded subtraction problem with two digit numbers the strategy was no longer successful and used appropriately. (Rose, numeracy report, p.5)

Lisa was *making connections between procedures* as well as demonstrating *knowledge of terminology*. Lisa's developing MCK related to more than one dimension of the Knowledge Quartet. Engaging in these planning and assessment practices enabled Lisa to begin to develop a teacher identity and to realise the importance of *foundation knowledge*.

As a comparison, during year 2, Rose was bringing deeper insights of her developing *foundation knowledge* and *connections* to demonstrate her emerging teacher identity. Rose was developing her *foundation knowledge* and *awareness of purpose* as her understanding of mathematics changed; extending her *foundation knowledge* by *using terminology* and *making connections between procedures and concepts* when describing how Molly used her fingers during a numeracy interview when considering how children learn. She was also *making connection between procedures* such as area and volume when attending lectures and tutorials.

Primary mathematics practicum experiences

Practicum experiences (shown in Table 2) are reported in this section. The programme provided placements in primary schools during year 1, year 2 and year 4. In year 3, Lisa and Rose completed a placement in a secondary school, but neither observed or participated in secondary mathematics lessons. Of the 144 days in placement, 102 days were undertaken in primary schools.

Year-1 practicum

In year 1, PSTs completed 15 school visits during the semester, including a week (5 days) during the second semester. An excerpt of Lisa's and Rose's reflections on year 1 practicum is shown in Table 8.

Both Lisa and Rose completed year-1 practicum with students in the early years, and neither were responsible for planning and teaching a mathematics lesson. They helped small groups or individual students during mathematics classes. We infer that these experiences may have assisted with conceptual understanding of mathematics and *foundation knowledge*, making *connections* by noticing the different topics within the mathematics curriculum during their weekly visits. However, insufficient data were collected to justify this inference.

Year-2 practicum

In year 2, PSTs completed 32 days in schools during the semester, including a 2-week block placement, with a programme requirement to plan and teach at least 20 primary

Table 8 Reflection on teaching experience (year-1 practicum)

Lisa	Rose
I was in a year 1. We did really basic maths. I just taught small groups and I never did any whole class.	I was in a year 1/2 class. I just kind of helped them and I didn't teach a maths lesson. We focused more on literacy in first year.

Table 9 Reflection on teaching experience (year-2 practicum)

Lisa	Rose
I have done all of my maths lessons with year 3 students. It got to the stage I wasn't writing up lesson plans and I would come in and my mentor would just say do something like that. Today the lesson was about subtraction. It was a game testing their subtraction skills. My mentor gave me a lesson from a year 3 maths book and I thought of my own way to do the lesson. Before the lesson I read Booker [textbook] on subtraction to get some ideas. I didn't find anything difficult… For the lesson, I needed to know different ways they might subtract. My mentor is helpful and always giving me books to look up including maths.	I feel like I can take away more maths stuff this year which is good [because] I am in a year 6 class. My mentor would help me plan ... I would like you to do angles and she would go away and I would think about how I would cover what she wants me to do then I would present it to her [for feedback]. Today was the first lesson I taught and I was shaking. The main mathematical ideas were naming angles, using a protractor properly, estimating different angles. Before the lesson I had to go away and look up different websites. The lesson was about recognition of angles between lines particularly when lines are parallel or perpendicular. I needed to refresh the definitions for the lesson and now I think I know them off by heart.

mathematics lessons. Lisa completed her practicum with a year-3 class and Rose experienced a year-5 and year-6 class (Table 9).

Lisa had some opportunities to *make connections* when planning; however, this may have been restricted because she was not required to write a lesson plan. When planning a lesson, Lisa was making *choice of examples* and *decisions about sequencing*, *transformation* and *connection*. Her choice and examples were limited because she relied on questions from a textbook and focused on subtraction procedures during the lesson rather than helping students learn by representing the mathematics. Extending or deepening her *foundation knowledge*, *transformation* and *connection* was therefore limited. Whilst recognising the need to know different ways children could subtract, she *adhered to the textbook*.

In comparison, Rose commented that she continued to carefully plan and revise before teaching. By planning lessons in detail, Rose would be more likely to rehearse the questions she might ask the students, check mathematical terms and how to complete the mathematics she planned to teach, therefore providing greater opportunity to reflect on and develop her *foundation knowledge*, *transformations* and *connections*. Both mentor teachers helped when planning lessons; however, Rose would have had greater opportunity to extend her *foundation knowledge*, *connections* and *transformation* when planning because she prepared a lesson plan that she shared with her mentor for feedback. Both Lisa's and Rose's teacher identity seemed to be influenced or supported by their mentor teachers' practices.

Year-4 practicum teaching experiences

In year 4, Lisa and Rose completed 50 days of practicum teaching experience, both were in composite year 3/4 classes at different schools. Lisa and Rose provided reflections on their experiences (Table 10).

Lisa had inadequate opportunities and experiences teaching mathematics during her final block placement because a Masters-level student from another university taught many of the mathematics lessons. Again, she did not write a lesson plan. Whilst she

Table 10 Reflection on teaching experience (year-4 practicum)

Lisa	Rose
This year I am in a double classroom and there is another pre-service teacher who is taking a lot of the maths lesson so I just help out. When I planned the maths lesson for today I met with my mentor. She gave the children a pre-test. I used this information to assess the children. My mentor teacher said she wanted me to cover height, width, length, perimeter and area. ... I didn't do a detailed lesson plan I just write in the class planner for the week. I made up the sheet with the questions for the children to answer. For this lesson, I knew what area and perimeter were but usually I have to and I will get onto Google and refresh other vocabulary that I missed out.	Each week before I go, I talk with my mentor about the maths lesson I will teach the following week. She gives me some ideas then I go home and plan the rest. I try to email it a couple of days before so she can check it. In my lesson plan, I included definitions of the terms required for the lesson, such as, equilateral, reflex angle. I still use my learning log when planning and writing my lesson plans.

claimed to analyse the formative assessment data, Lisa *adhered to the textbook* to plan a worksheet for questions' topics identified by her mentor teacher. She also needed to check and "refresh" her *overt subject knowledge*. These experiences illustrate that Lisa had limited opportunity to extend categories of the Knowledge Quartet and remained dependent on her mentor teacher for direction rather than developing an independent mathematics teacher identity.

Rose's reflections demonstrate her ability to extend her knowledge for teaching by *making connections between concepts* and *procedures* when writing a lesson plan and teaching and reflecting on her planning with her mentor teacher. Her lesson plan also showed evidence of extending her *foundation knowledge* such as developing *terminology* and *making decisions about sequencing a lesson*. Continuing to record notes in her mathematics learning log also assisted Rose to check and revise her *foundation knowledge*.

Table 11 Lesson observation (year-4 practicum)

Lisa	Rose
Lisa was observed teaching a lesson on perimeter and area of regular quadrilaterals. Lisa introduced the lesson by asking the students questions: What do we use a ruler for… what are the terms we use when measuring in metric measurement… Width what does it mean… Lisa drew a rectangle on the board then asked the students to help her label H for height, W for width. She then said, "If this square is two centimetres high and 4 centimetres wide what is the equation I am using to write the area… When you find an area of a shape you times the height by the width." Lisa had designed a worksheet for the students to complete for the remainder of the lesson. Students were asked to work in pairs and find objects in the room such as a tissue box to measure the area.	Rose introduced her lesson by asking the students to brainstorm what was similar and different about a set of laminated triangles she had made. Next, students sorted and labelled the triangles into three groups whilst discussing properties of scalene, isosceles or equilateral triangles. When asking the students questions Rose asked, "What do these triangles have in common… how could we categorise them or put them into sub groups... turn and talk to the person next to and ask them what they think… which one is an equilateral…" Rose had designed a worksheet for the students to complete. They had to measure the lengths of the triangles and label them as equilateral, isosceles or scalene.

Year 4 lesson observation

Lisa and Rose were observed teaching a lesson with year-3 and year-4 students in the fourth year of their programme. Lisa taught a lesson related to measurement including perimeter and area, whilst Rose explored different triangles with her students. Table 11 shows some of the field notes recorded by the first author.

Lisa did not have a lesson plan although she did say she had planned the lesson with her mentor. When teaching, Lisa was able to make *decisions about sequencing* her lesson and *make connections between concepts* but relied on and taught *procedural knowledge* when teaching area. She chose closed questions limiting opportunities to *deviate from the agenda* and respond to *contingencies*.

Prior to teaching the observed lesson, Rose had planned the lesson with her mentor and prepared a detailed lesson plan. She also continued to use her mathematics learning log when planning her lessons. Rose was able to draw on her *foundation knowledge*, *make decisions about sequencing* her lesson, *make connections between concepts* and used triangles to help the students discuss their properties—coded as *transformation* and *choice of examples*. By choosing to use open questions, Rose was demonstrating how she could rely on her MCK and that she was prepared for *contingencies*.

Discussion

Over the duration of the course, Lisa and Rose developed their MCK for teaching. When analysing and reporting on opportunities and influences that enhanced the development of Lisa's and Rose's MCK, there was more evidence of opportunities to develop their *foundation knowledge* and to make *connections*. Data that provided evidence of developing *transformation* and *contingency* knowledge were limited, perhaps because of the nature of the data collection. Even though they were enrolled in the same units and both had extended practicum opportunities, their learning of MCK for teaching was quite different. Lisa was able to make connections to sequence tasks within lessons and between lessons but *adhered to the textbook* and relied on her mentor teacher for these resources. She needed to revise terminology, rules and procedures for each new topic. Rose showed an ongoing commitment to learning by deepening her *foundation knowledge*, evidence included her sustained use of a learning log. She documented her plans for lessons and reviewed these with her mentor teacher before teaching. She demonstrated more evidence of developing *transformations* through her selection of examples and valuing of physical representations for making sense of the mathematics. Factors that contributed or hindered development of MCK included similarities and differences in Lisa's and Rose's learner identity, teacher identity, sustained engagement in mathematics learning and quality of teaching and learning experiences, including practicum experiences. These four factors are discussed next.

Learner identity

Lisa and Rose had similar pre-programme identities including negative experiences when learning mathematics during their own schooling (Table 4). When commencing teacher education, they may have believed that mathematics is difficult to learn because of these negative experiences. However, before teaching, Lisa and Rose needed to develop their *foundation knowledge* for teaching, and it is fundamental to the other three dimensions of the Knowledge Quartet when helping students to learn mathematics (Rowland et al. 2009).

During year 1 and year 2, the PSTs completed practicum placement in primary schools (Table 5). Lisa and Rose both may have had difficulty relying on their *foundation knowledge* when assisting students during practicum in year 1. Therefore, it was important to assist PSTs to identify their weaknesses early in their programme, such as completing a MCSK test.

Of concern was that Lisa may have continued to have difficulty during year 2 and year 3 when helping students learn mathematics because of her difficulty knowing the mathematics she might be expected to teach. Whereas, Rose was able to rely on her learner identity to extend her *foundation knowledge*, and she passed the MCSK test earlier in the course than Lisa.

As a comparison in year 1 (Table 5), Lisa's investigation most likely only provided revision of her *foundation knowledge* rather than meeting her personal learning goals of extending her MCK including statistics, decimals and fractions. In comparison, Rose was able to demonstrate greater self-efficacy in her ability to extend her *foundation knowledge* because her assignment showed calculations including multiplication of decimal numbers. Possibly, the year-1 assignment was too open-ended for Lisa and further guidance would have supported and motivated her to do better and to demonstrate understanding of mathematics content that she would be required to teach in the primary years related to statistics.

Rose's programme experiences may have assisted her actions that helped her make choices and want to extend her *foundation knowledge*. She was able to identify her weakness in her *foundation knowledge* as preparation for passing the MCSK test after her first attempt in year 1. In doing so, she chose to use a mathematical learning log to document and develop her *foundation knowledge*. She continued to use this resource and strategy throughout the course.

Other studies have also highlighted the impact of PSTs' attitudes toward mathematics and their learner identity. Mathematics anxiety impacts on PSTs' studies and the way they perceive themselves as learners of mathematics (Wilson 2015). Whilst Young-Loveridge et al. (2012) agreed that helping PSTs to address their misconceptions and develop positive attitudes of mathematics will assist them to become more confident teachers. As teacher educators, we need to carefully consider course design to address PSTs' limited MCK plus pedagogical change. Programme design should focus on developing more professional and proficient mathematics teachers (Klein 2012; Young-Loveridge et al. 2012).

Teacher identity

Both coursework and practicum experiences provided opportunities to develop teacher identity, *foundation knowledge*, *transformation*, *contingency* and *connections*. However, Lisa and Rose responded to these opportunities differently. For example, when considering the categories of *foundation knowledge*, Rose showed evidence of *awareness of purpose* because she could apply her knowledge of MCK and PCK when passing the MCSK test in year 1. Rose planned in detail, revising her MCK for teaching during placement, demonstrating her teacher identity. Lisa did not maximise her opportunities, by consolidating, learning, extending or applying her MCK in order to pass her MCSK test in year 1 or when planning and teaching mathematics lessons in year 2 and year 4.

Practicum experiences including the school year level the PSTs experienced as well as expectations of the mentor teacher may have influenced teacher identify. Since practicum in year 1 mostly focused on planning and teaching literacy (Table 8), both had limited experiences to identify themselves as mathematics teachers. In year 2 (Table 9), Lisa had the opportunity to consolidate her *foundation knowledge* for teaching in the early years. In contrast, because Rose experienced year-6 students, she needed to extend her MCK and *foundation knowledge* for teaching by planning and teaching more difficult content. Both experienced years 3 and 4 during the fourth year of their programme.

Transformation was evident more so with Rose when planning to teach and by her *choice of examples* and *representations* such as using a protractor when teaching geometry. When making *connections*, Rose's planning would have also helped her with her *decisions about sequencing* as well as *conceptual appropriateness* when she classified triangles with students. Whereas, Lisa's teaching and *foundation knowledge adhered to the textbook* and *concentrated on procedures* when teaching measurement. Lisa's choice of closed question types and limited open-ended type questions restricted her opportunities for promoting *contingencies*.

Lisa was not given the opportunity to teach in year 5 or 6 during the practicum. Therefore, Lisa's limited *breadth* of experiences during placement would have impacted on her ability to extend her *foundation knowledge* and *make connections* for teaching higher year levels. The expectations of Lisa's mentor teachers may have impacted on her teacher identity as she was not expected to write lesson plans, whereas Rose wrote detailed plans and discussed these with her mentor teacher before teaching. By writing detailed lesson plans, Rose continued to develop her skills to think like a teacher, *making choices of tasks*, *sequencing the lesson* and listing her questions and *mathematical vocabulary* for her lessons. Furthermore, Rose continued to record and use her mathematical learning log that she started in year 1 to help revise and consolidate her *foundation knowledge*, including *subject matter knowledge*.

Kazemi et al. (2009) recommended that PSTs must be encouraged to rehearse their teaching if they are to enact ambitious mathematics instruction. Helping PSTs to make connections with their teacher identity and what they learn at university and teaching may also include observation of model lessons (Livy and Downton 2018) or discussion of student work samples (Livy et al. 2017). Grootenboer (2008) reminded us that learning to teach also involves emotions, attitudes and beliefs and that teacher educators should not ignore these if PTSs are to adopt new mathematical beliefs.

Sustained engagement

PSTs' opportunities to learn during teacher education programmes can be shaped by the programme structure and approaches (McDonald et al. 2014; Tatto et al. 2009). In this study, the results identified that the structure and approaches of coursework and practicum contributed to PSTs' development of categories of the Knowledge Quartet. However, sustained engagement for each year of the programme was a factor that may have restricted PSTs' opportunities to maximise development of their mathematics teacher identity. In particular, the programme did not provide PSTs with the opportunity to revise their knowledge for teaching primary mathematics during the second half of their programme. When reviewing this programme these will be important factors to consider and since this study have now been addressed by including a year-4 mathematics education unit into the programme.

Quality of teaching and learning experiences

Although this programme provided many days in primary schools, the quality of teaching and learning experiences may have influenced PSTs' development of MCK. Mentor teachers have the opportunity to play a vital role in shaping PSTs' MCK when planning lessons, observing and providing feedback to PSTs. Limited data of these interactions were collected, as part of this study, therefore, cannot be reported in detail. However, this study did identify the different expectations of mentor teachers in particular regarding emphasis of lesson planning, which was likely to have impacted on PSTs' ability to develop the skills and knowledge for teaching mathematics.

Further research is needed, but a recommendation of this study would be to ensure that PSTs are planning lessons in detail to maximise their opportunities to develop their MCK. Mentor teachers should also be provided with professional learning to introduce them to the Knowledge Quartet as a tool for supporting PSTs when planning, teaching and reflecting after teaching. The expectation to reflect on practice and opportunities afforded by reflection was not evident in the data collected for this study. Turner (2012) agreed that focused reflection is important for supporting the development of MCK.

Conclusion

These findings contribute to the literature by reporting on a longitudinal study of the opportunities and influences that enhanced the development of PSTs' MCK taking into account both coursework and practicum. Multiple data were reported, rather than focusing on only one aspect or short-term programme experiences, demonstrating the usefulness of the Knowledge Quartet when reporting on the many programme experiences. Previously, the Knowledge Quartet has mostly been used as a tool with teachers and PSTs when reflecting and observing lessons. In this study, the Knowledge Quartet was used to highlight the consequences of programme structure in order to help pre-service teachers to establish and sustain a positive mathematics learner identity, build teacher identity and develop MCK.

Although Lisa's and Rose's programme structure was less common, including primary and secondary teacher education, the findings are helpful when considering future teacher education programme design. Identifying programme structures that influence how PSTs develop their knowledge for teaching is needed (Tatto et al. 2012). Recommendations from this study would include familiarising lecturers and mentor teachers with the dimensions of the Knowledge Quartet as a tool for focusing on critical aspects of what PSTs need to learn to support the development of their MCK. Consideration of the consequences of programme structure is also important when designing, redesigning or revising programmes in order to help PSTs to establish and sustain a positive mathematics learner identity, build teacher identity and develop MCK. Implications for future programme design would be to recommend all PSTs experience teaching year 6 as early as possible during practicum and that coursework experiences in year 4 are introduced to help PSTs to sustain the development of their MCK before graduating. Although there is current policy designed to ensure PSTs enter teacher education with numeracy skills (LANTITE assessment (ACER 2015)), there is still a need to ensure all primary teacher education programmes assist PSTs to provide many and varied opportunities that extend their MCK. A limitation of the study was that data reported on one programme. Further studies may report on other teacher education programmes so as to contribute to the findings of this study and to identify further opportunities and influences that may improve PSTs' MCK.

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